Ph.D. PRELIMINARY EXAMINATION

MICROECONOMIC THEORY

Applied Economics Graduate Program

September 2021

The time limit for this exam is four hours. The exam has four sections. Each section includes two questions. Answer one question from each section. Before turning in your exam, number each page of your answers in sequential order, and identify the question (for example, III.2 for section III and question 2). Indicate, by circling below, the questions you completed. If you answer more than one question in a section and do not circle a question corresponding to that section, the first of the two that appears in your solution paper will be graded.

****

STUDENT ID LETTER: _______ (Fill in your code letter)

Answer one question from each section. Indicate the one you answered by circling:

Section I:	Question 1	Question 2
Section II:	Question 1	Question 2
Section III:	Question 1	Question 2
Section IV:	Question 1	Question 2

***** TURN IN THIS SHEET WITH YOUR ANSWER PAGES*****

Part I

Answer at most one question from Part I.

Question I.1

Consider the linear expenditure system functional form for a demand system. For simplicity, there are only 3 goods. Expenditure on good i is:

$$p_i x_i = p_i \gamma_i + \beta_i (w - \sum_{k=1}^{3} p_k \gamma_k), \text{ with } \sum_{k=1}^{3} \beta_k = 1 \quad \text{for } i = 1, 2, 3$$

a) What is the Walrasian demand for good 1? [Hint: This is very easy and can be done in a few seconds.]

b) Use your answer to part a) and the Slutsky equation to obtain the derivative of the Hicksian demand for good 1 with respect to the price of good 2. You can denote this as $\partial h_1(p, u)/\partial p_2$. You should be able to simplify this expression to some extent.

c) You want to use estimates of the linear expenditure system to test the Slutsky symmetry condition. More specifically, you want to check whether $\partial h_1(p, u)/\partial p_2$ equals $\partial h_2(p, u)/\partial p_1$. To get started, what is the expression for $\partial h_2(p, u)/\partial p_1$ for the linear expenditure system with 3 goods?

d) Given your answers to b) and c), what restriction or restrictions will you test regarding the β and γ parameters of the linear expenditure system to test whether Slutsky symmetry holds? What does this tell you about the usefulness of the linear expenditure system?

Note: The remaining parts of this question, parts e) and f), do not depend on the answers to parts a) through d), so if you have trouble with parts a) through d) you can still work on parts e) and f).

e) Recall that the linear expenditure system has the following expenditure function:

$$e(p, u) = \sum_{k=1}^{3} p_k \gamma_k + u \prod_{k=1}^{3} p_k^{\beta_k},$$

where p (without a subscript) is a vector of prices. Work out the indirect utility function that corresponds to this expenditure function, using w to denote wealth (this is very simple algebra). For the rest of this question, assume that $\beta_1 = 0$, $\beta_2 = 0.5$, $\beta_3 = 0.5$, $\gamma_1 = 1$, $\gamma_2 = 2$ and $\gamma_3 = 3$. What is the value of utility for a consumer who has a wealth (w) of 100 and faces the following prices for the 3 goods: $p_1^0 = 5$, $p_2^0 = 4$ and $p_3^0 = 9$? Do not worry about the "0" superscript on the prices; it is used in part f).

f) Finally, suppose that prices increase to the following (note the 1 superscript): $p_1^1 = 6$, $p_2^1 = 9$ and $p_3^1 = 16$. This vector of prices can be denoted by p^1. What is the compensating variation (CV) for this consumer for this price change? State in words what CV is measuring.

Question I.2

Risk Aversion and Certainty Equivalence. Consider two Bernoulli utility functions for an amount of money, x, where $x > 0$:

$$u_1(x) = 1 - x^{-1}$$

$$u_2(x) = x(1 + x)^{-1}$$

a) For the first Bernoulli utility function, $u_1(x) = 1 - x^{-1}$, check whether it is increasing in x for all $x > 0$. If so, is it also strictly increasing? Also, check whether it is concave, strictly concave, or not concave for all $x > 0$.

b) For the second Bernoulli utility function, $u_2(x) = x(1 + x)^{-1}$, check whether it is increasing in x for all $x > 0$. If so, is it also strictly increasing? Also, check whether it is concave, strictly concave, or not concave for all $x > 0$. [Hint: the first derivative can be simplified to something very simple, and doing so will make the second derivative easier to calculate.]

c) For both of these Bernoulli utility functions, calculate the Arrow-Pratt coefficient of **absolute** risk aversion. According to this criterion, which Bernoulli utility function has more absolute risk aversion? If you need to condition on something to answer this question, please do so.

d) For both of these Bernoulli utility functions, calculate the Arrow-Pratt coefficient of **relative** risk aversion. According to this criterion, which Bernoulli utility function has more relative risk aversion? If you need to condition on something to answer this question, please do so.

e) Consider a person with the first utility function, $u_1(x) = 1 - x^{-1}$. Suppose that this person faces the following lottery:

$$\text{Prob}[x = 3] = 0.5$$

$$\text{Prob}[x = 6] = 0.5$$

What amount of money is the "certainty equivalent" for this person for this lottery? You should be able to simplify your answer to give a round number.

f) Last, consider another "lottery" for this person. Let x be a random number drawn between 3 and 6. That is, x follows a uniform distribution between 3 and 6, so that $f(x) = 1/3$ if $3 \leq x \leq 6$, and 0 for $x < 3$ and $x > 6$. For the same Bernoulli utility function as in part e), what is the "certainty equivalent" of this lottery? How does this answer compare to your answer for part e), and what is the intuition for this comparison? **Hint:** To answer this you will need to do a little integral calculus, and note that $\log(6) \approx 1.79$, $\log(3) \approx 1.10$ and $1/0.23 \approx 4.35$.

Part II

Answer at most one question from Part II.

Question II.1

Consider the production possibility set (**PPS**):

$$\mathbf{PPS} = \left\{(\mathbf{q}, -\mathbf{z}) \in \mathbb{R}_+^2 \times \mathbb{R}_-^2 : {z_1}^{\alpha}{z_2}^{\beta} \geq \frac{{q_1}^2 + {q_2}^2}{2}\right\}$$

where z_1 and z_2 are inputs, q_1 and q_2 are outputs, and $\alpha > 0$ and $\beta > 0$ are constant parameters. This **PPS** is nonempty, strictly convex, closed, and satisfies weak free disposal of output and inputs.

a) Under what additional conditions on α and β will this PPS exhibit non-decreasing returns to scale. Justify your answer.

b) Derive the input distance function assuming $q_1 > 0$ or $q_2 > 0$, **and** also derive the marginal rate of technical substitution between z_1 and z_2 using this input distance function.

c) The cost function for this **PPS** is $C(r_1, r_2, q_1, q_2) = \Omega {r_1}^{\frac{\alpha}{\alpha+\beta}} {r_2}^{\frac{\beta}{\alpha+\beta}} \left(\frac{{q_1}^2+{q_2}^2}{2}\right)$ where $r_1 > 0$ and $r_2 > 0$ are input prices, and $\Omega > 0$ is a constant.

   i) Using this cost function, set up the profit maximization problem where the competitive price of q_1 is $p_1 > 0$. Next, assume that the producer has a monopoly with q_2, where the inverse demand for q_2 is $p_2 = 100 - \frac{q_2}{2}$. Set up the producer's profit maximization problem and derive the first order conditions assuming the solution is interior.

   ii) Find the unconditional supplies for q_1 and q_2 using the first-order conditions you derived in Part c) i).

   iii) Use the unconditional supplies from Part c) ii) and duality results to find the unconditional demand for z_1. ***Note: You do not need to simplify your answer.***

Question II.2

Suppose that a memory chip manufacture has learned that the country it exports to is considering imposing a tariff on memory chips. Unfortunately, the manufacture must decide how many memory chips to produce before it learns if the tariff will in fact be imposed. Therefore, the manufacture faces two possible states of the world, which are labeled $s = N$ if no tariff is imposed and $s = T$ if a tariff is imposed. The manufacture's cost function is $c(q)$ where $q \geq 0$ is the number of memory chips it produces. These costs are increasing at an increasing rate: $c'(q) > 0$ and $c''(q) > 0$. With a competitive price of memory chips equal to p and tariff equal to t, the manufacture's state contingent profits are

$$\pi^N(q) = pq - c(q) \text{ and } \pi^T(q) = (p - t)q - c(q).$$

Let $W(\pi^N, \pi^T)$ be the manufacture's state contingent utility of profit with $W_s(\pi^N, \pi^T) = \frac{\partial W(\pi^N, \pi^T)}{\partial \pi^s} > 0$ for $s = N, T$. Also assume the manufacture is risk averse such that $W_{ss}(\pi^N, \pi^T) = \frac{\partial^2 W(\pi^N, \pi^T)}{\partial \pi^{s2}} < 0$ for $s = N, T$.

a) Set up the manufacture's utility maximization problem and derive its first order condition for an interior solution that can be solved for q.

b) If the manufacture finds out whether there will be a tariff before choosing its output, it's state contingent profits are

$$\pi^N(q^N) = pq^N - c(q^N) \text{ and } \pi^T(q^T) = (p - t)q^T - c(q^T)$$

where $q^N \geq 0$ is output with no tariff and $q^T \geq 0$ is output with a tariff. Set up the manufacture's utility maximization problem and derive its first order conditions for an interior solution that can be solved for q^N and q^T if this is the case instead.

c) Which of the three outputs is the largest: q, q^N, or q^T? Which of the three outputs is the smallest: q, q^N, or q^T? Justify your responses. ***Hint: compare first order conditions from parts a) and b)***.

d) Would your answer to part c) change if the manufacture was not risk averse (i.e., $W_{ss}(\pi^N, \pi^T) \geq 0$ for $s = N, T$)? Explain.

e) If the country's goal is to maximize tariff revenue, should it let the manufacture know it has decided to impose the tariff before or after the manufacture's output is selected? Explain. ***Hint: Take advantage of your answer to part c).***

Part III

Answer at most one question from Part III.

Question III.1

Assume that there are $n \geq 3$ firms in a sector, with an inverse demand function given by

$$p(Q) = 20 - Q$$

where $Q = \sum_{j=1}^{n} q_j$ and the production cost for firm i is $c_i q_i$, with $c_i = 4$ for all firms.

(a) Find the symmetric Nash (Cournot) equilibrium when firms simultaneously choose output quantities.

(b) Let firm 1 be the market leader. First firm 1 chooses its output, then the other $n - 1$ firms simultaneously choose their output. Find the subgame perfect Nash equilibrium in this case.

(c) Conversely, assume that firm 1 is the last mover, and responds to the simultaneous choices of all other firms. Find the subgame perfect Nash equilibrium.

(d) Compare the profitability of firm 1 and some other arbitrary firm j across these three scenarios. Explain the intuition behind your result.

Question III.2

Consider the two-player extensive form game in the figure below.

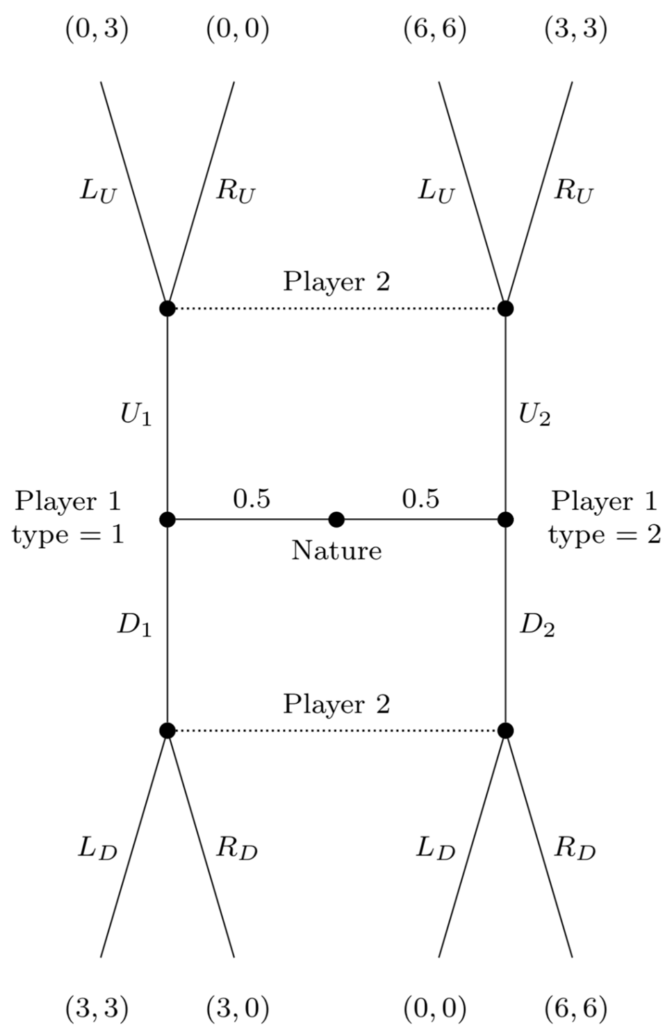
(a) What is each player's pure strategy set?

(b) Construct the normal form game for this figure.

(c) Find all of the pure strategy Nash equilibria in this normal form game.

(d) Find all of the pure strategy (separating and pooling) perfect Bayesian equilibria for this extensive form game. Justify your answer(s).

(e) Compare the equilibrium strategies found in parts (c) and (d), explain the intuition behind your findings.



Part IV

Answer at most one question from Part IV.

Question IV.1

Consider a 2×2 competitive exchange economy, where subscript $j = 1, 2$ indexes consumers. Goods are x and y, and preferences are represented by the utility functions

$$U_1(x_1, y_1) = x_1 y_1 \qquad \text{and} \qquad U_2(x_2, y_2) = \min\{x_2, 2y_2\}.$$

The aggregate endowment for the economy is $\omega = (8, 6)$.

(a) In a carefully labeled Edgeworth box diagram draw at least two indifference curves for each consumer. Also draw the contract curve: the set of Pareto-optimal allocations.

(b) Now suppose initial endowment vectors are $\omega_1 = (4, 6)$ and $\omega_2 = (4, 0)$. Derive the demands functions for the two consumers. Be sure to include demand behavior when prices are zero. You do not need to draw offer curves in your diagram.

(c) Derive all Walrasian equibrium price-allocation vectors (p^*, x^*, y^*). Indicate and label the equilibrium (or equilibria), both quantities and the relative prices (that is, a budget line through ω), in your Edgeworth box from part a.

(d) Now suppose the initial endowment vectors are $\omega_1 = (0, 1)$ and $\omega_2 = (8, 5)$. Preferences are still those given above. Once again find all Walrasian equilibria for the economy.

(e) Consider the allocation $x_1 = 4$, $y_1 = 4$, $x_2 = 4$, and $y_2 = 2$. *True or False:* this allocation is Pareto optimal. If *True*, find a redistributed endowment vector ω' that can support this allocation as a Walrasian equilibrium. Find the associated equilibrium price vector. If *False*, find a feasible allocation that Pareto dominates the given allocation.

Question IV.2

Consider the usual social-choice setting with a finite set of m alternatives X and a finite set of individuals $J = \{1, \dots n\}$. Preferences P_j are strict on X and we say that a profile is $\{P_j\}_{j=1}^n$. **Answer part (a) and (b), and also answer *either* (c) or (d).**

(a) State Arrow's theorem. Include definitions of each of the four axioms (independence of irrelevant alternatives; unrestricted domain; the Pareto principle; and dictatorial) as well as the social welfare function at issue.

(b) For each of the three axioms other than dictatorial, explain what a violation would mean for a social welfare function. Tell whether you think each is a reasonable requirement for collective decision making, and why or why not.

Answer *either* (c) or (d)

(c) Provide a proof of step 2 of the first proof of Arrow's Theorem. That is, take as given a person j who, from step 1, is semi-decisive (SD) over some pair a, b. Show that this person j is decisive over all $x, y \in X$ (that is, j is a dictator).

(d) Take as given a person j^* who, from steps 2 and 3 of the second version of the proof, due to Geanakoplos, is extremely pivotal (that is, by unilaterally changing his vote, j can move some alternative from the bottom of the social ranking to the top). Provide a proof of steps 3 and 4 of the second proof of Arrow's Theorem. Recall that step 3 establishes that the extremely pivotal voter is decisive on all pairs not involving alternative c, which was moved from bottom to top of each voter's rankings in step 2. Step 4 establishes that this voter is also decisive on any pair involving c.